

Stabilization of tokamak plasma by lithium streams

Leonid E. Zakharov,

Princeton University, Princeton Plasma Physics Laboratory

Presented to APS 2000 MHD meeting

October 22, 2000, Quebec, Canada

OUTLINE

1. Basics of liquid lithium MHD.
2. Flow pattern of magnetic propulsion.
3. Theory of stabilization.
4. Flow locked mode.
5. Prospects for high beta.

1 Basics of liquid lithium MHD

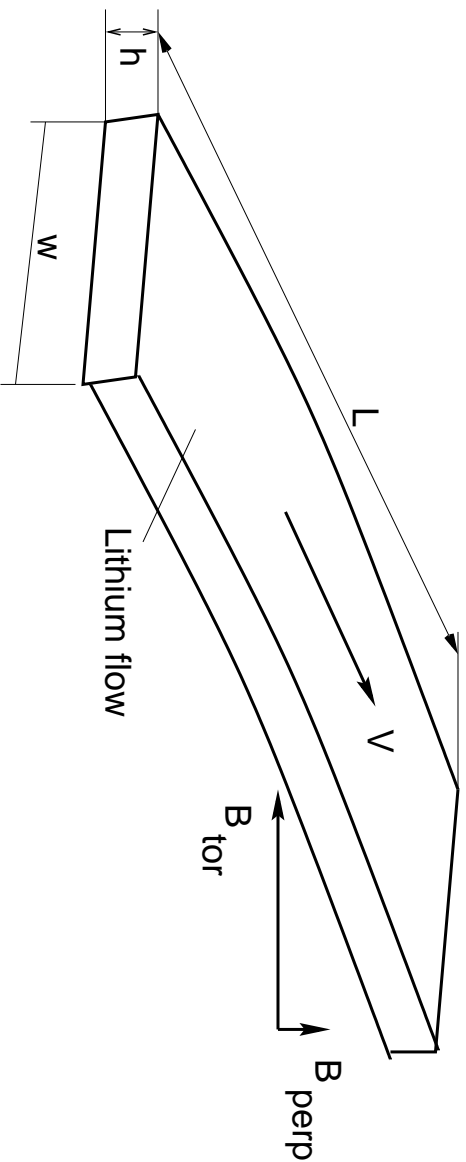
There 3 magnetic Reynolds numbers which control lithium MHD in toka-maks

dynamics : $\mathfrak{R}_0 \equiv \mu_0 \sigma LV$,

electro-dynamics : $\mathfrak{R}_1 \equiv \mu_0 \sigma hV$,

dynamics : $\mathfrak{R}_2 \equiv \mu_0 \sigma \frac{h^2}{L} V$, (1.1)

$$\mu_0 \sigma \simeq 4 \left[\frac{\text{sec}}{m^2} \right].$$



Characteristic flow parameters:

$$\begin{aligned} V &= 20 \text{ m/sec} \rightarrow \rho \frac{V^2}{2} \simeq 1 \text{ [atm]}, \\ B &= 1 \text{ T} \rightarrow \frac{B^2}{2\mu_0} = 4 \text{ [atm]}, \\ B &= 5 \text{ T} \rightarrow \frac{B^2}{2\mu_0} = 100 \text{ [atm]}. \end{aligned} \tag{1.2}$$

Dynamic pressure losses are determined by \Re_0 and \Re_2

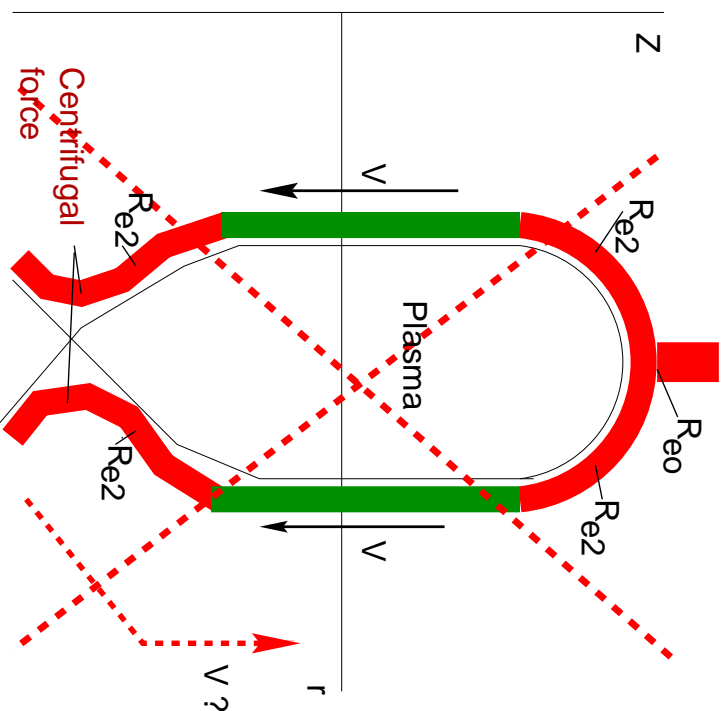
$$\begin{aligned} \Re_0 : \quad \Delta \rho \frac{V^2}{2} &= \mu_0 \sigma L V \frac{B_{\perp}^2}{2\mu_0}, \\ \Re_2 : \quad \Delta \rho \frac{V^2}{2} &= \mu_0 \sigma \frac{h^2}{L} V \Delta \frac{B_{\parallel}^2}{2\mu_0}, \\ \mu_0 \sigma &\simeq 4 \left[\frac{\text{sec}}{m^2} \right]. \end{aligned} \tag{1.3}$$

Magnetic fields from the currents in the stream are determined by \Re_1

$$\Re_1 : \quad B_{out} - B_{in} = \mu_0 \sigma h V B_{\perp}. \tag{1.4}$$

Lithium “water-falls” are **incompatible at the basic level** with the tokamak strong toroidal field

$$\begin{aligned}
 h &= 0.1 \text{ m}, & L_1 &\simeq 0.2 \text{ m}, & L_2 &\simeq 3 \text{ m}, & V &> 2 - 5 \text{ [m/sec]}, \\
 \Re_0 &= 4L_1 V \Rightarrow 1.6, \\
 \Re_2 &= 4\frac{h^2}{L_2} V = 4\frac{h}{L_2} (hV) \simeq 0.01 - 0.025.
 \end{aligned}
 \tag{1.5}$$

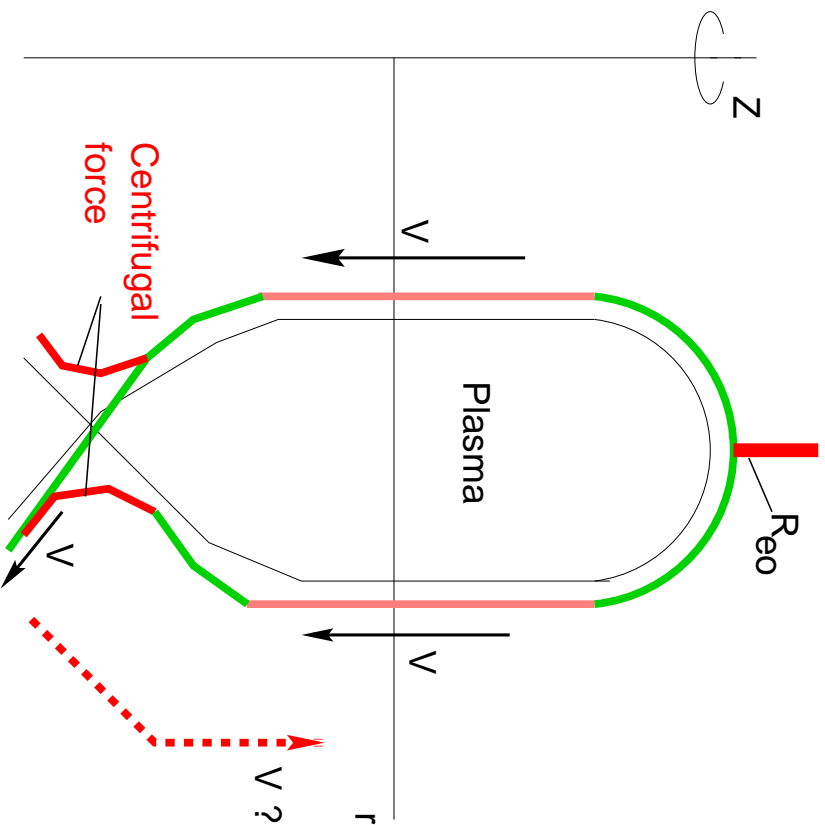


$$\rho \frac{V^2}{2} \ll \Re_2 \Delta \frac{B_{tor}^2}{2\mu_0}$$

Momentum driven thin walls have a lot of unresolved problems in lithium MHD

$$h = 0.01 \text{ m}, \quad L_1 \simeq 0.02 \text{ m}, \quad L_2 \simeq 3 \text{ m}, \quad V \simeq 20 \text{ [m/sec]},$$

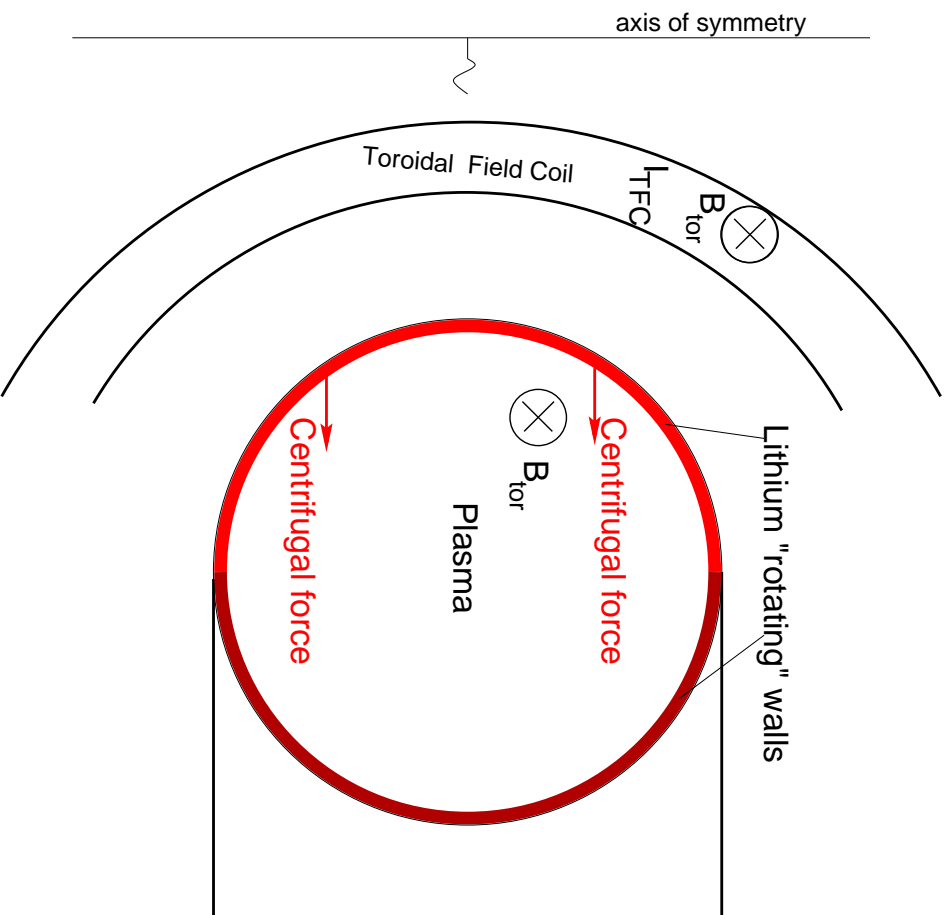
$$\Re_2 = 4 \frac{h^2}{L_2} V \simeq 1.3 \cdot 10^{-4}. \quad (1.6)$$



$$\Re_0 = 1.6, \quad \rho \frac{V^2}{2} < \Re_0 \frac{B_{pol}^2}{2\mu_0}$$

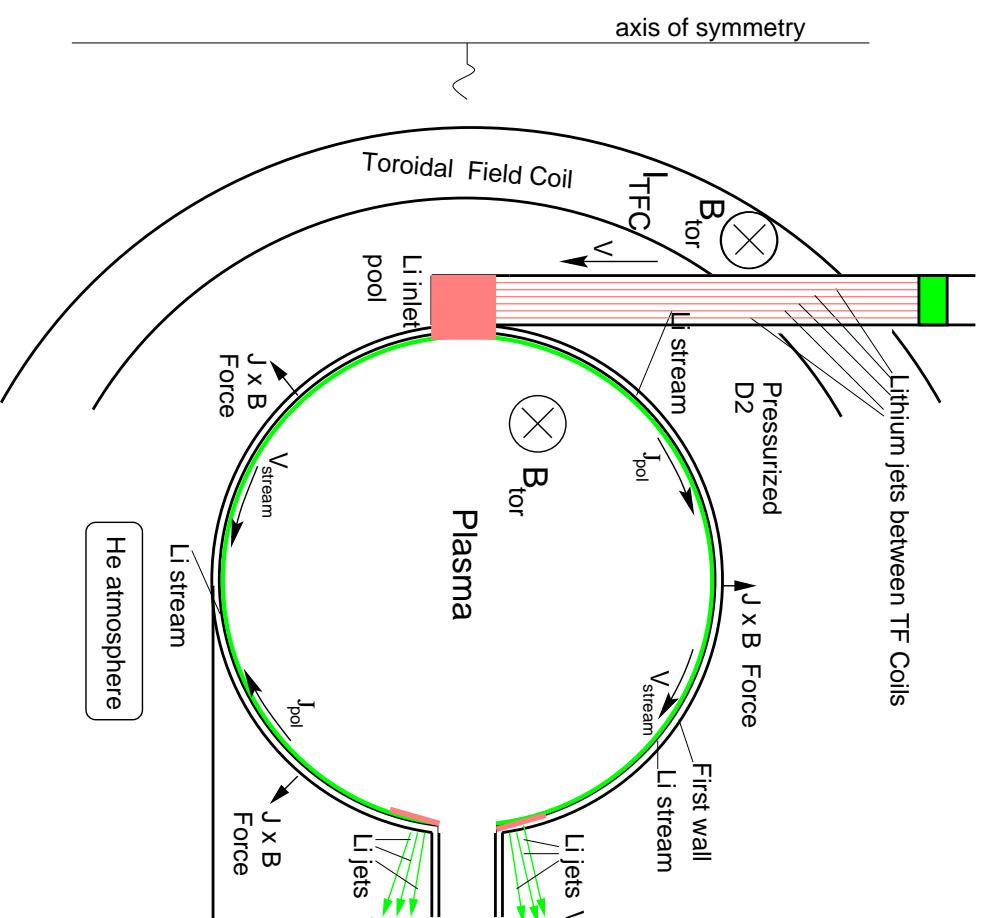
2 Flow pattern of magnetic propulsion

“Rotating” liquid lithium walls are incompatible with tokamaks.



- no inlet/outlet
- poloidal rotation damps as $R\rho \frac{dV}{dt} \simeq \Re_2 \frac{B_{tor}^2}{2\mu_0}$
- toroidal rotation is impossible due to centrifugal force.

Magnetic propulsion makes MHD of intense lithium streams compatible with tokamaks.



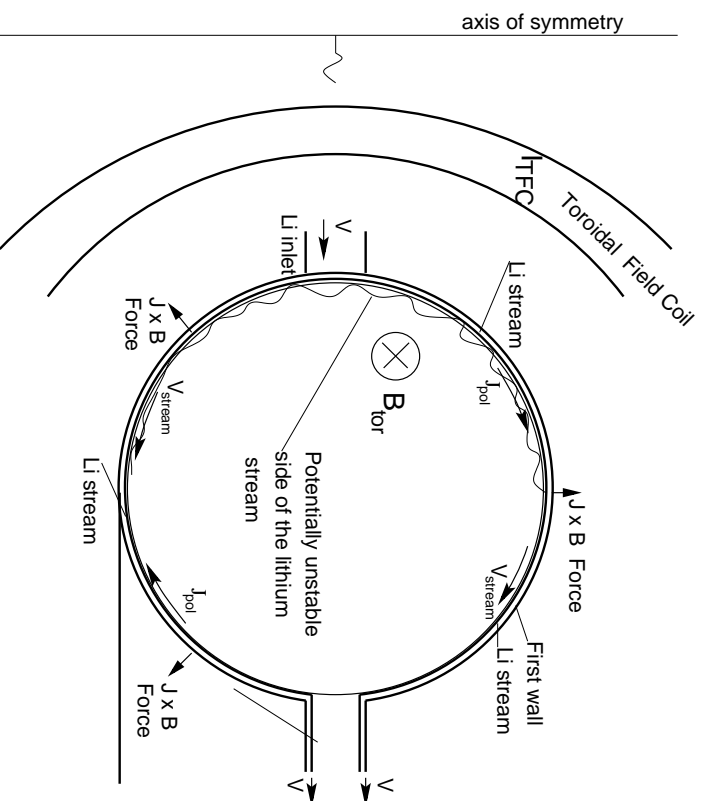
Driving pressure

$$p_{wall|outlet} > 1 \text{ atm}, \quad p_{wall|inlet} - p_{wall|outlet} > 1 \text{ atm.} \quad (2.1)$$

Flow parameters

$$V \simeq 20 \text{ m/sec}, \quad h \simeq 0.01 \text{ m}, \quad \Re_1 = 0.8. \quad (2.2)$$

Stream are robustly stable due to centrifugal force



$$\rho \frac{\langle V^2 \rangle}{2} > \frac{a}{2R} p_{wall} n_r$$

3 Theory of stabilization

Flow pattern of magnetic propulsion eliminates the possibility of mode locking into the conducting wall (“Rotating” walls do nothing).

The theory includes an arbitrary geometry of the guide wall

$$\begin{aligned}\mu_0 \tilde{\mathbf{j}}_w &\equiv \frac{\nabla \rho \times \nabla I}{\delta a} = -\frac{I'_\varphi}{J_w} \mathbf{e}_\theta + \frac{I'_\theta}{J_w} \mathbf{e}_\varphi, \\ J_w &\equiv r_w(\theta) h(\theta) \sqrt{g_{\theta\theta}}, \quad g_{\theta\theta} = (r_w')^2_\theta + (z_w')^2_\theta.\end{aligned}\tag{3.1}$$

It links the electric current in the streams with parameters which can be extracted from existing numerical codes

$$(\mathbf{D}_v - \mathbf{D}_p) \vec{\psi}(a) = -i M \vec{I}.\tag{3.2}$$

It formulates the equation for electric current in the streams and leads to a dispersion relation for the growth rate

$$(\mathbf{D}_v - \mathbf{D}_p) \vec{\psi} = \mu_0 h \sigma \gamma (\mathbf{M} \mathbf{S}^{-1} \mathbf{M}) \vec{\psi} + i \mathfrak{R}_{\mathbf{L}_1} (\mathbf{M} \mathbf{S}^{-1} \mathbf{V} \mathbf{M}) \vec{\psi}.\tag{3.3}$$

Dispersion relation for the cylindrical case

$$a\Delta'_m\psi_m = \tau_{res}\gamma\psi_m + \Re_1 \sum_k (m + 2k + 1) v_{2k+1}^* \psi_{m+2k+1}, \quad (3.4)$$

where

$$\tau_{res} = \mu_0 \sigma h a, \quad v_{2k+1} = \frac{2}{i\pi(2k+1)}, \quad v_{2k} = 0. \quad (3.5)$$

There is coupling with a nearest sitelite modes and then, with each second satellite mode.

Three harmonics approximation immediately shows the stabilizing effect

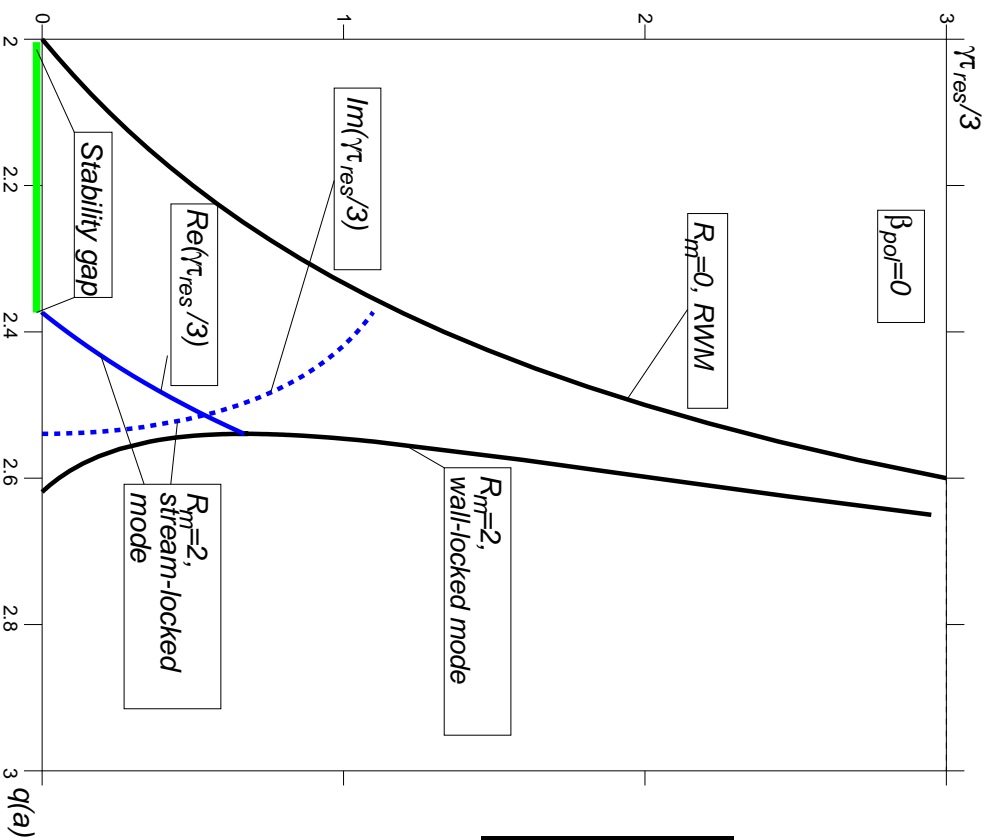
$$a\Delta'_m - \tau_{res}\gamma + \frac{m(m+1)\Re_1^2|v_1|^2}{a\Delta'_{m+1} - \tau_{res}\gamma} + \frac{m(m-1)\Re_1^2|v_1|^2}{a\Delta'_{m-1} - \tau_{res}\gamma} = 0. \quad (3.6)$$

It also shows possibility for the mode, which is locked into one of streams.

4 Flow locked mode

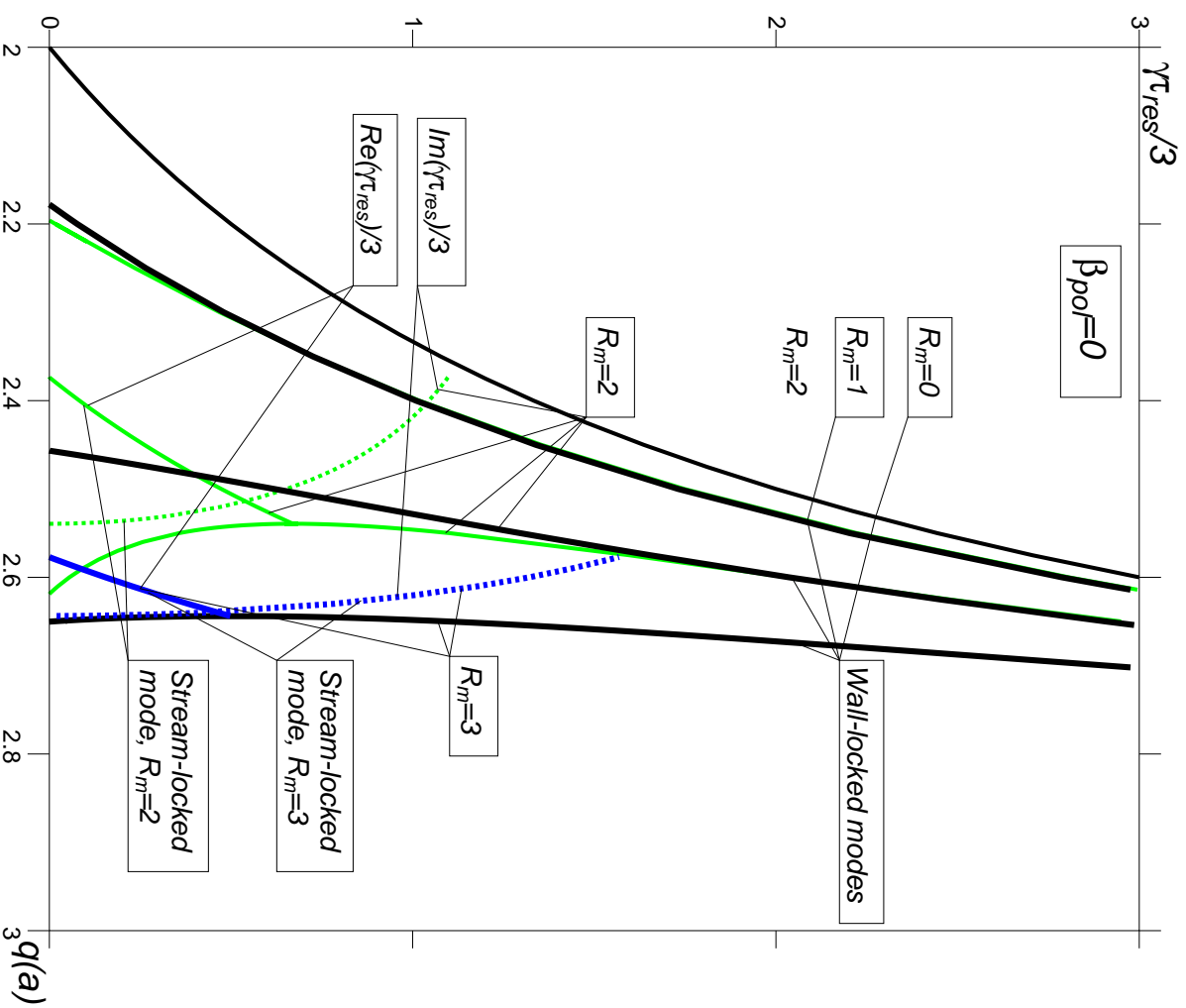
Resistive wall mode is well affected by the flow.

Flow-locked mode determines limits of stabilization.

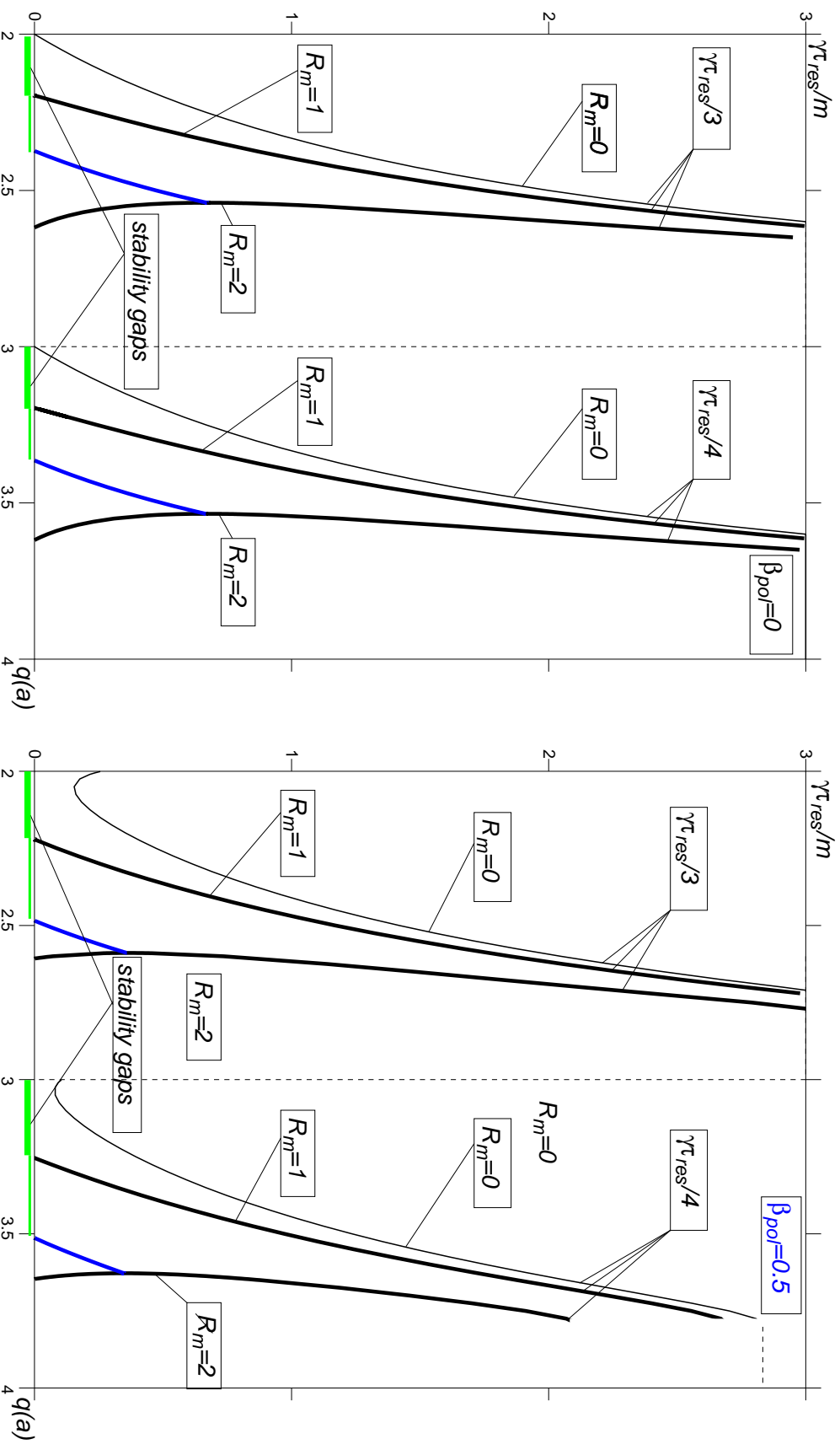


$$\frac{a\Delta'_m - \tau_{res}\gamma}{a\Delta'_{m-1} - \tau_{res}\gamma} + \frac{m(m+1)\Re_1^2|v_1|^2}{a\Delta'_{m+1} - \tau_{res}\gamma} + \frac{m(m-1)\Re_1^2|v_1|^2}{a\Delta'_{m-1} - \tau_{res}\gamma} = 0$$

Comparison of 3 harmonics vs full matrix calculations.



Stability gaps are insensitive to m-number. Finite β can be stabilized.



5 Prospects for high beta

Intense lithium streams move the conducting wall surface to the plasma boundary.

This would represent the major effect on stability, allowing feedback system close to the plasma surface.

In addition, the streams themselves contribute to MHD stabilization.

In combination with an expected low-recycling regime and flattened temperature and current density profile, this would open the path to high betas even in circular machines.